P425/1
PURE
MATHEMATICS
Paper 1
July / August 2023
3 hours

MASAKA DIOCESAN EXAMINATIONS BOARD
Uganda Advanced Certificate of Education
Joint Mock Examinations 2023
PURE MATHEMATICS
Paper 1
3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** questions from section **B**.*

*All necessary working **must** be shown clearly.*

Graph paper is provided.

Silent non-programmable calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 Marks)

1. Given that $\log_3 x = p$ and $\log_{18} x = q$ show that $\log_6 3 = \frac{q}{p-q}$ (05 marks)

2. The first, second, third and nth term of a series are 4, -3, -16 and $(an^2 + bn + c)$ respectively. Find the values of a, b and c. (05 marks)

3. The sum of the height and radius of a right circular cone is 9cm. Show that the maximum volume of the cone will be $36\pi cm^3$. (05 marks)

4. If $(y-2)^2 = 4(x-3)$ is a parabola state its vertex, focus and equation of the directrix. (05 marks)

5. Solve for x, $2\sin^2\left(\frac{x}{2}\right) - \cos x + 1 = 0$ where $0 \leq x \leq 2\pi$. (05 marks)

6. The surface area of a sphere is decreasing at a rate of $0.9m^2/s$. When the radius is 0.6m. Find the rate of change of the volume of the sphere at this instant. (05 marks)

7. Find the shortest distance of the point A(1, 6, 3) from the line
   $r = \mathbf{i} + \mathbf{j} + \mathbf{k} + \beta(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (05 marks)

8. Find $\int x \sin^{-1} x \, dx$. (05 marks)

SECTION B: (60 Marks)

9. Prove that the equation of the tangent to the parabola $y^2 = 4ax$ at the point P(ap=2+, 2ap) is $py = x + ap^2$. The tangent at P meets the axis at Q. Given that F is a fixed point (h, 0) and R is a point such that QFRP is a parallelogram. Find;
   i) the coordinate of R.
   ii) the equation of the locus of R. (12 marks

10. (a) If α and β are roots of the equation $ax^2 + bx + c = 0$. Find the value of $\alpha^2 - \beta^2$ in terms of a, b and c.

    (b) MADDO gives 5% compound interest per annum to its members. Mukwatampola deposits shs. 1,000,000 at the beginning of every year starting with 2023. How much will he collect at the end of 2030 if he does not make any withdrawals within this period? (07 marks)

11. If $Z = \frac{(2-i)(5+12i)}{(1-2i)^2}$

a) Find the;

(i) modulus of Z.

(ii) argument of Z. (08 marks)

b) Represent z on argand diagram.

c) Write z in a polar form. (04 marks)

12. Given the curve $f(x) = \frac{2x^3-x^2-25x-12}{x^3-x^2-5x+5}$

a) Find;

(i) the value of x for which $f(x) = 0$

(ii) the asymptotes for f(x)

(iii) x and f(x) intercepts for the curve.

b) Hence sketch the curve. (12 marks)

13. a) Given that L_1 is the line $\mathbf{r_1} = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and L_2 is the line $\mathbf{r_2} = \begin{pmatrix} -3 \\ 7 \\ p \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$

If L_1 and L_2 intersect. Find the values of P and the point of intersection. (06 marks)

b) Find the equation of the plane passing through the points A(1, 1, 0) B(3, -1, 1) C(-1, 0, 2) and find the shortest distance of the point (3, 2, 1) to the plane (06 marks)

14. a) Prove that for any triangle ABC.

$$\frac{a^2-b^2}{c^2} = \frac{\sin(A-B)}{\sin(A+B)}$$

b) Express $\sqrt{5}\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$. Hence solve the equation $\sqrt{5}\cos x + 2\sin x = 1.2$ for $0^\circ \leq x \leq 360^\circ$.

(06 marks)

15. Given that $\frac{x^2-8x+5}{(2x+1)(x^2+9)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+9}$

Find the values of A, B and C.

Hence show that $\int_0^3 \frac{x^2-8x+5}{(2x+1)(x^2+9)}\,dx = \frac{1}{2}\ln 7 - \frac{\pi}{3}$ (12 marks)

16. a) Solve the differential equation:

$$(x+1)\frac{dy}{dx} - 3y = (x+1)^4 \text{ given } y(1) = 6$$

b) Mukasa walks to school at a speed proportional to the square root of the distance he still has to cover. If he covers 900m in 10 minutes and the school is 2500m from his home. Find how long he takes to get to school.

(07 marks)

END